Resources Management. A Simple Example of Optimization Problem.

Your company makes two products – **A** and **B**. The capacity of your manufacturing unit is 525 hours per week. It takes five hours to produce one **A**, and three hours to produce one **B**.

Labor hours per unit	
A	**B**
5 hours	3 hours

Capacity = 525 hours

Your customer demand for **B** is unlimited. They will buy as much as you can produce. **A**, on the other hand, have a maximum demand of only 100 per week.

Your suppliers can only provide you with the raw materials needed to produce a maximum of 75 **B** per week. The raw materials for **A** are freely available.

The profit per unit, or unit contribution (C) is $2.50 for **A**, and $2.00 for **B**.

How many units of **A** and **B** should you produce each week to maximize your profit?

1. Express the Problem Mathematically

Maximize:

C = 2.5A + 2B (maximum profit)

Subject to these capacity constraints:

$5A + 3B \leq 525$ (capacity constraint),

$A \leq 100$ (**A** sales constraint),

$B \leq 75$ (**B** raw materials constraint)

$A \geq 0, B \geq 0.$

2. Graph the Problem

When you have two variables (our example has two product combinations), you can draw up a graph to show the solution. This is a great tool for understanding the question in its entirety. With **A** production on the x-axis, and **B** production on the y-axis, your graph can show the maximum production capacities for all possible combinations of **A** and **B**.

a) Draw the capacity line: The end points of the capacity line are the maximum possible **A** and **B** that can be produced. You can produce: 105 **A** (525/5) and 0 **B** (100,0) or 0 **A** (0,175) and 175 **B** (525/3) The capacity line joins these two points. The corresponding straight line equation is

$$5A + 3B = 525$$

The Windows Calculator (Graphic variant) for illustration can be recommended. Figure 1 illustrates a graph of this straight line. Figure 2 illustrates a graph of the half-plane defined by this straight line.

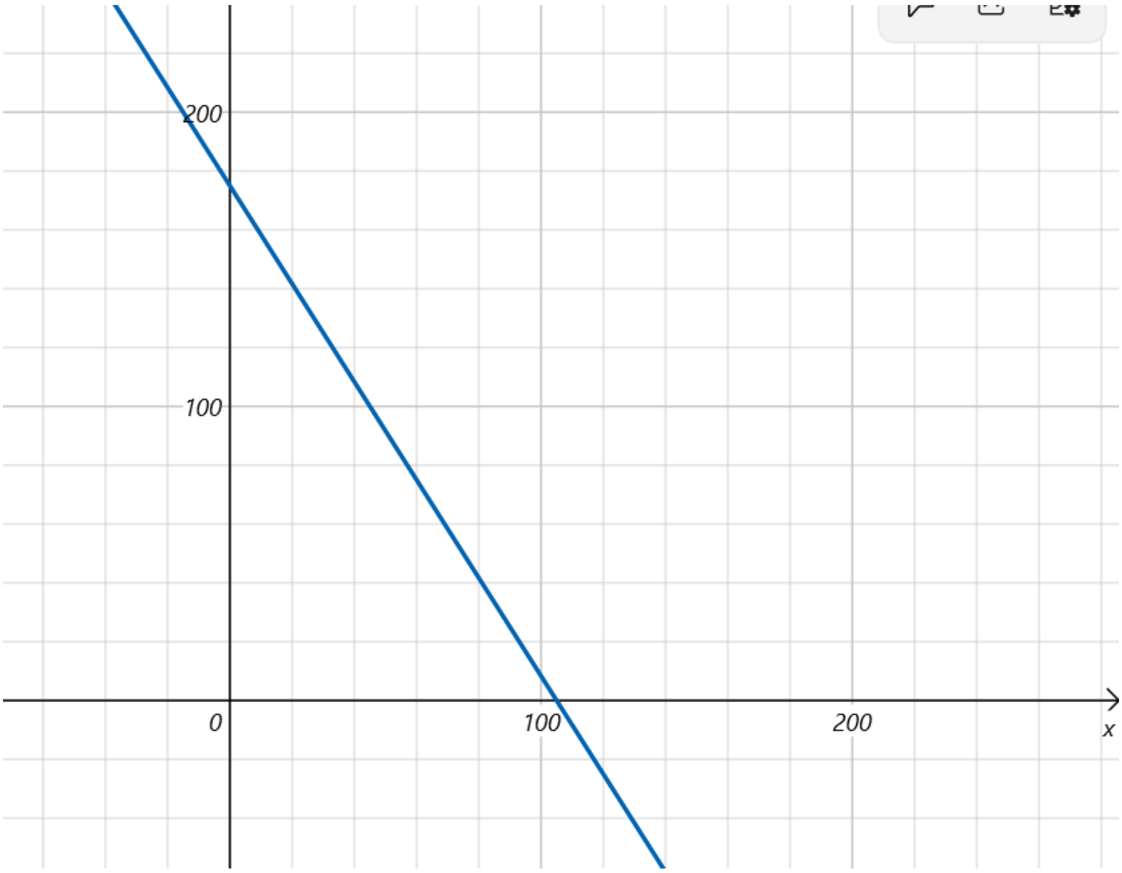

Figure 1

Figure 2

b) Draw the sales constraint line for **A** ($A \leq 100$).

c) Draw the raw materials constraint for **B** ($B \leq 75$).

These three constraint lines are shown below as straight lines (Figure 3) or as an intersection of half-planes (Figure 4).

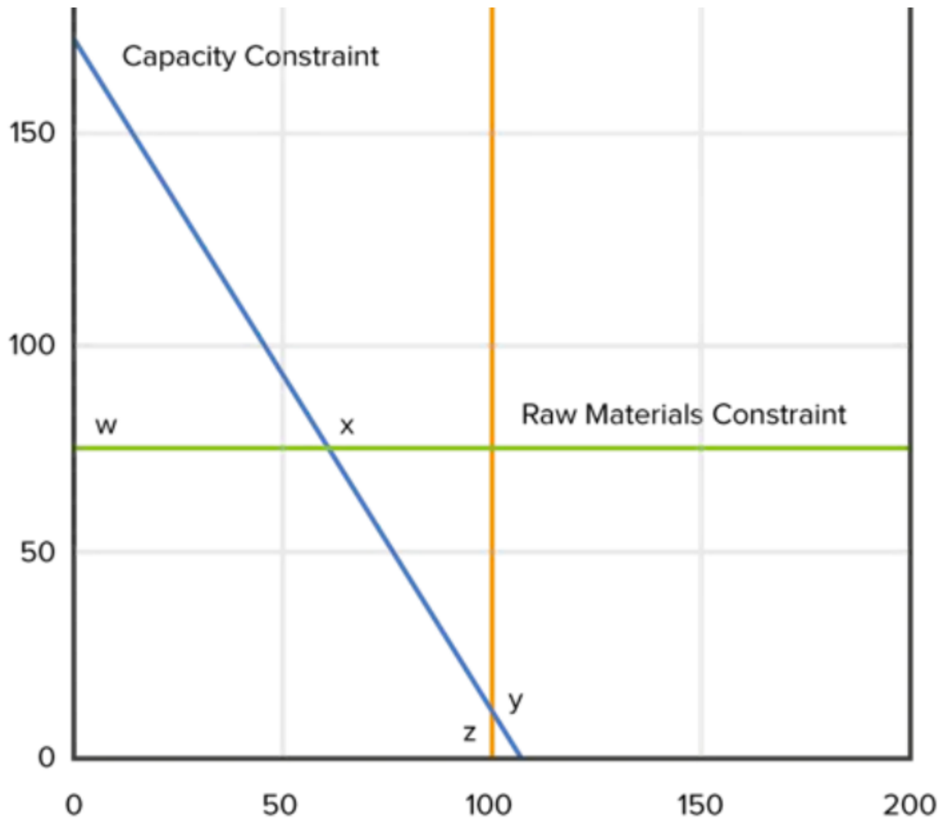

Figure 3

Figure 4

The area bounded by the three constraint lines and the x and y axs is the set of possible combinations of **A** and **B** production. Anything beyond those borders is not possible – given these constraints, and assuming that production of **A** and **B** is greater than 0.

The optimal combination will be somewhere along the outside border. Anything inside the boundary would not use all of the available capacity.

3. Calculate the Optimal Value

How do you know where on the boundary the optimization point is? It will be at one of the intersections of the various constraint lines. In this example, there are four intersection points (w, x, y, and z). To determine the exact point of optimization, you examine each of these. As you already know that points w and z represent the extreme points used in the constraint lines, you can consider the calculations below.

1. **Maximum contribution (C) at point w A = 0, B = 75,**

C = 2.5A + 2B (profit) C = (2.5 x 0) + (2 x 75) **= 150.**

2. **Maximum contribution (C) at point *x***

5A + 3B = 525, B = 75 Substitute the value of B in the first equation, and solve for A: 5A + (3 x 75) = 525,
5A = 300, that is A = 60.
So at point x the production is 60 **A** and 75 **B**:
C = 2.5A + 2B (maximum profit), C = (2.5 x 60) + (2 x 75), that is
C = 300.

3. **Maximum contribution (C) at point *y***

5A + 3B = 525, A = 100.
Substitute A = 100 into the first equation: (5 x 100) +3B = 525.
3B = 25. B = 8 (round down because you cannot sell part of **B**).
C = 2.5A + 2B (maximum profit). C = (2.5 x 100) + (2 x 8) **= 266.**

4. **Maximum contribution (C) at point z A= 100, B = 0.**

C = 2.5A + 2B (maximum profit). C = (2.5 x 100) + (2 x 0) **= 250.**

*Based on these calculations, the optimal production for **A** and **B** is at point x (60 **A** and 75 **B**) with a maximal profit 300$.*

This is a simple ***linear programming*** example.
Linear programming (linear optimization) is an optimization technique for a system of linear constraints and a linear objective function. An objective function defines the quantity to be optimized, and the goal of linear programming is to find the values of the variables that maximize or minimize the objective function.

In reality, most business problems involve so many variables and constraints that you wouldn't (or couldn't) try a manual solution. Linear programming software programs (for example, MS Excel) can solve the equations quickly and easily, and they provide a great deal of information about the various points within the possible set. You can also run "what if" scenarios to determine things such as which additional machinery to buy, or whether to add an extra shift of workers, resources, etc.

Key Points

Linear programming uses a mathematical or graphical technique to find the optimal way to use limited resources.

When you have a problem that involves a variety of resource constraints, linear programming can generate the best possible solution. Whether it's maximizing things like profit or space, or minimizing factors like cost and waste, using this tool is a quick and efficient way to structure the problem, and find a solution.